Reliability Modelling of Phased Mission Multi-State Systems via a Scenario Inference Method

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Outline

1. Background and Introduction
2. FST Models for Failure Mechanism Dependence
3. FST Models for Multi-state Systems
4. FST Models for Phased-mission Systems
5. Case Study and Conclusion
Part 1

Background and Introduction
Background and Introduction

Data-based methods (Black box methods)

Life tests

Life distribution

Reliability

Problems:

• How about it is very hard to get enough samples to be tested to get enough data about the product life?
• How about the product enjoys a pretty good quality which means you have to spend a quite long time (maybe the whole life) to get the data, even using the accelerating technology?
• How about the data is not reliable or not correct?
PoF methods (White box methods)

Generalized Damage Parameter, GDP

\[ N = c(\Delta W_s) \frac{1}{2} \left( \frac{V_t}{V_c} \right)^{1-k} \left( \frac{V_c}{V_t} \right)^n \]

For a complicated system, a single model is too weak to describe the process of the system failure, for the failure not only has the relationship with one single mechanism or several independent mechanisms, also relies on the correlations among the failure mechanism, system structure and so on. For the multi-state systems and phased-mission systems, the situation even more complex.
Generate a system model

Petri Net model just for a simple component
Generate a system model

We have a dream!

- Can we generate a model automatically?
- Can we get the failure path of the system automatically?

The first step:

We need to draw a map!
Part 2

FST Models for Failure Mechanism Dependence
Every possibility of the failure is shown in a failure scenario tree, which means the FST can draw all possible path from very beginning to the very end (failure or some certain event). Such characteristic is the reason that the FST can be the base of automatic modelling.

\[
F(t) = 1 - \prod_{i=1}^{n} \left[ 1 - \int_{0}^{t} f_i(\sigma) d\sigma \right]
\]
FST Models for Failure Mechanism Dependence

2.2 Trigger

2.3

2.4

Failure Mechanism Tree

Failure Scenario Tree

Failure probability of the system

\[ F(t) = 1 - \left[ 1 - \int_0^t f_a(\sigma) \, d\sigma \right] \prod_{i=1}^n \left[ 1 - \int_0^{t-T_C} f_i(\sigma) \, d\sigma \right] \]

- Before \( C_1 \), just \( M_a \)
- After \( C_1 \), \( M_a + \{M_1 \ldots M_n\} \) then compete
- In the FST, \( \triangle \) is like a switch:
  - \( C_1 \) happens – turn on
  - \( C_1 \) not happen – turn off

Trigger Event

MACT

\( M_1 \ldots M_n \)

\( M_a \)

\( C_1 \)
FST Models for Failure Mechanism Dependence

2.3 Acceleration and Inhibition

MACC / MINH

Failure Mechanism Tree

C₁
Trigger Event

Failure probability of the system

\[ F(t) = 1 - \prod_{i=1}^{n} \left[ 1 - \int_{0}^{t-T_C} f_{ri}(\sigma) d\sigma \right] \]

the time of the change

Failure Scenario Tree

Failure distribution function of Mᵢ after acceleration or inhibition
2.4 Accumulation

Accumulating Rule:

\[ \zeta = \frac{X_\text{th}}{\Delta X} \]

The damage threshold of the system

The unit damage of system

\[ \Delta X = \sum_{i=1}^{n} \frac{\lambda_i \Delta X_i}{\lambda_i} \]

The unit damage due to \( M_i \)

A scaling factor of \( M_i \)

The failure time of the system

The failure time due to \( M_i \)

Failure probability of the system

\[ F(t) = P \left( \frac{1}{\sum_{i=1}^{n} \frac{\lambda_i}{t_i}} \leq t \right) \]
FST Models for Multi-state Systems
3.1 Time Order FST

- The Time Order FST is used to generate model for multi-state components.
- A switch between two states refers to a stage. (3 states – 2 stages)
- The terminal event represent multiple states, rather than just one result (like ‘failure’ in the binary system)
Fault order FST is used on the level of component and subsystem, which means the event in the FST is the state of the component, rather than the failure mechanism dependence.

- three components here
- three states per component
  1. operation (X-1)
  2. degradation (X-2)
  3. fault (X-3)
(X: A, B or C)

Three-state systems
3.2 Fault Order FST

- Only the change of the state will be shown in the FST, in order to condense the size of the FST.
- Fault (or state-change) order of the components needs to be determined before drawing the Fault Order FST.
- The relationship between the state of the system and its components needs to be determined.

\[ S-1: A-1 \cap B-1 \]
\[ S-2: A-2 \cap B-1 \text{ or } A-2 \cap B-2 \]
\[ S-3: A-2 \cap B-3 \text{ or } A-3 \]
3.2 Fault Order FST

parallel

---

2/3 system

---
Part 4

FST Models for Phased-mission Systems
4.1 Event Order FST

These phases in a mission enjoy a certain order.

4.2 Event order FST is also called event order scenario tree (EOST).

A mission with two phases:

![Diagram of a mission with two phases](image)

**X-S:** succeed

**X-F:** fail

The event order FST for a mission with two phases:
4.1 4.2 Multi-state EOST

• In terms to a phased-mission multi-state system, it could be under different states in the same phase or mission.

• The EOST, which shows the binary state, is upgraded into MS-EOST to draw the multiple state.

\[ \text{Ph}_i \text{-} j \text{ or } \text{MS}_i \text{-} j \text{ means the system in phase } i \text{ or the current mission is at state } j \]

\[ \begin{align*}
\text{MS}_1 &: \text{Ph}_1 \text{-} 1 \cap \text{Ph}_2 \text{-} 1 \\
\text{MS}_2 &: \text{Ph}_1 \text{-} 1 \cap \text{Ph}_2 \text{-} 2, \text{Ph}_1 \text{-} 2 \cap \text{Ph}_2 \text{-} 1 \\
\text{MS}_3 &: \text{Ph}_1 \text{-} 1 \cap \text{Ph}_2 \text{-} 3, \text{Ph}_1 \text{-} 2 \cap \text{Ph}_2 \text{-} 3, \text{Ph}_1 \text{-} 3
\end{align*} \]
Case Study and Conclusion
An electrical system, which is regarded as a PM-MSS, is required to perform a mission with four phases. The performing order and the duration of each phase are shown as:

<table>
<thead>
<tr>
<th>Phase</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000h</td>
</tr>
<tr>
<td>2</td>
<td>3000h</td>
</tr>
<tr>
<td>3</td>
<td>2600h</td>
</tr>
<tr>
<td>4</td>
<td>2400h</td>
</tr>
</tbody>
</table>

![Diagram](image-url)
### State Definition

<table>
<thead>
<tr>
<th>Phase 1, 2, 4</th>
<th>Phase 3</th>
<th>Mission</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State 1</strong></td>
<td>V is in state 1 and at least two of three ICs are in state 1.</td>
<td>All components are in state 1.</td>
</tr>
<tr>
<td><strong>State 2</strong></td>
<td>V is in state 2 and no more than one IC is in state 3. Or V is in state 1 and at least one of the two best functioning ICs among three ICs is in state 2.</td>
<td>At least one of the components is in state 2 and none is in state 3.</td>
</tr>
<tr>
<td><strong>State 3</strong></td>
<td>V is in state 3 or at least two of three ICs are in state 3.</td>
<td>At least one of the components is in state 3.</td>
</tr>
</tbody>
</table>
### Failure Mechanism and Correlation

<table>
<thead>
<tr>
<th>Component</th>
<th>Mechanism</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Crack</td>
<td>Trigger by shock</td>
</tr>
<tr>
<td>VF</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td>IC₁</td>
<td>TDDB</td>
<td>Accumulation</td>
</tr>
<tr>
<td></td>
<td>NTBI</td>
<td>/</td>
</tr>
<tr>
<td>IC₂</td>
<td>Creep</td>
<td>Acceleration</td>
</tr>
<tr>
<td></td>
<td>EM</td>
<td>/</td>
</tr>
<tr>
<td>IC₃</td>
<td>VF</td>
<td>Accumulation</td>
</tr>
<tr>
<td></td>
<td>TF</td>
<td>/</td>
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<tr>
<td></td>
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</tr>
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</table>

**V** vibration fatigue  
**TF** thermal fatigue  
**TDDB** time-dependent dielectric breakdown  
**NBTI** negative bias temperature instability  
**EM** electrical migration
Time Order FST for every component

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<table>
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<tr>
<th>Level</th>
<th>Failure Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-1</td>
<td>The state of V has never changed, so neither Crack nor VF has effect on V.</td>
</tr>
<tr>
<td>V-2</td>
<td>Due to the continuing influence of VF, it is possible that V change its state from V-1 to V-2.</td>
</tr>
<tr>
<td>V-3</td>
<td>(1) no Crack</td>
</tr>
<tr>
<td></td>
<td>Based on the state V-2, the continuing influence of VF can change the state into V-3.</td>
</tr>
<tr>
<td></td>
<td>(2) Crack occurs.</td>
</tr>
<tr>
<td></td>
<td>No matter when the Crack starts, V will change its state into V-3.</td>
</tr>
</tbody>
</table>
Fault Order FST for each structure

Phase 3

<table>
<thead>
<tr>
<th>Component</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC₁</td>
<td>State 1</td>
</tr>
<tr>
<td>IC₂</td>
<td>State 2</td>
</tr>
<tr>
<td>IC₃</td>
<td>State 3</td>
</tr>
</tbody>
</table>
Multi-state Event Order FST for mission

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</tr>
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<td>State 3</td>
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5.3 Calculating Results

- The value of reliability of the binary-state condition is larger than that of the multi-state condition.
- The probability of state 2 is generally increased first and then decreased. <not monotone>
- The sum of all state probabilities at the same time is always equal to 1.
- The state probability curve of a multi-phase system is not as smooth as that under single-phase condition, and an inflection point often occurs when phase changed.
- The evaluation of system reliability and state probability considering multi-state and multi-phase becomes closer to the engineering practice.
Thank you for listening!

Question Time!

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